

Program of the course AM110 – Mathematical Analysis part 1
(AA 2020-2021 -L Chierchia)

The set of real numbers and its main subsets:

- Sets, relations and functions.
- Axioms of real numbers.
- Elementary properties of ordered fields.
- Symmetric sets and functions. Absolute value and distance.
- Natural numbers. Subtraction in \mathbb{N} ; principle of well-ordering and its consequences.
- Sequences and recursion theorem (optional proof). Recursive definition of sums, products and powers.
- N^{th} powers, geometric sum and formula for $a^n - b^n$. Newton's binomial formula.
- Finite and infinite sets.
- Rational numbers. The rationals are countable. Gauss lemma.
- Least upper bound and greatest lower bound. Elementary consequences of the completeness axiom on integers.
- Roots. Powers with rational exponent.
- Monotone functions.

Theory of limits:

- The extended real system \mathbb{R}^* . Intervals and neighbourhoods.
- Internal, isolated, accumulation points. General definition of limit. Uniqueness of the limit.
- Sign permanence theorem. Comparison theorems.
- Side limits and monotone functions.
- Algebra of finite limits. Extended limit algebra.
- Some notable limits of sequences.
- The number of Nepero.
- Bridge theorem and characterisation of the sup/inf by sequences.
- Continuity: general considerations; theorem of existence of zeros. Intermediate value theorem.
- Classification of discontinuities.
- Limits for compound functions.
- Limits for inverse functions.
- A continuous and strictly monotone function on an interval admits a continuous inverse.
- Logarithms.
- Remarkable limits (exponential and logarithms).

Series:

- Numerical series: Elementary properties of series. Comparison criteria.
- Decimal expansions.
- Convergence criteria for series with positive terms.
- Criteria for series with real terms (Abel-Dirichlet, Leibniz).
- Exponential series. Irrationality of e . Speed of divergence of the harmonic series.
- Properties of trigonometric functions (in particular proof of the cosine addition theorem).
- Periodic functions. Monotonic properties of trigonometric functions.
- Inverse trigonometric functions.